The Nature of and System Inferences of Delay Distortion Due to Mode Conversion in Multimode Transmission Systems

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(Manuscript received July 1, 1963)

Quantitative estimates of delay distortion due to mode conversion in a multimode medium are made using an analysis based on modes coupled through power-transfer coefficients. This results in a simple translation from the spatial distribution of mode conversion to delay distortion without an intermediate step in the frequency domain. The expected value of the reconversion magnitude and its delay distribution relative to a driving impulse are found for (i) the case where the undesired mode loss is distributed (helix damped modes and higher-order circular electric modes) and (ii) the case where discrete mode filters are inserted (as in smooth-walled waveguide). Numerical estimates are given for TE_{01} *in 2-inch I.D. guides at 55 kmc.*

For both cases the power in the reconversion echo varies directly as the system length, and the shape of the echo is independent of length. For the case of distributed undesired mode loss the echo to impulse-excitation has an exponential shape in relative delay τ, *varying as* $e^{-\tau/\tau_0}$, *and for the case of partially absorbing mode filters the echo is a line-segment approximation to an exponential in* τ *(Fig. 4). The characteristic delay constant* τ_0 *is about 0.035 nanosecond for helix damped modes in an all-helix line, and is about 0.106 microsecond for* TE_{02} *in either helix or smooth-walled guide. For solid-walled guide with mode filters every 300 feet the characteristic delay constant (similar to* τ_0*) is about 2 nanoseconds.*

Estimates are made for signal interference effects from such echoes, taking account of the fact that the most limiting requirements on echoes in some system arrangements occur at $\tau \gg \tau_0$, *where the reconversion power is small. For PCM in smooth-walled copper waveguide with mode filters every 300 or 150 feet, it is concluded that pulse rates of 200 or 400 megabits might be used, with up to 20 or 40 miles respectively between regenerators;*

beat wavelength straightness variation mode conversion is controlling. For PCM in an all-helix waveguide, it is concluded that a pulse rate up to 5000 megabits and up to 746 miles between regenerators is permitted by mode conversion effects; diameter variations (TE_{02} conversion) are controlling.

For transmission of frequency division multiplex via a frequency-modulated carrier (FDM-FM), estimates based on the discrete-echo theory of Bennett, Curtis and Rice suggest that 4000-mile transmission of 2000 channel groups is possible in all-helix waveguide; diameter variations are controlling. An rms frequency deviation of about 15 mc (total band about 150 mc) would provide 40 db interchannel interference ratio at 900 miles, and larger deviations (σ) increase the allowed system length (z) according to $z \approx (\sigma)^{2.8}$. Even in solid-walled waveguide there is a good prospect for 4000-mile FDM-FM using guide tolerances already achievable.

Separate consideration is being given to delay distortion due to waveguide cutoff dispersion, which will be appreciable in some configurations described and will require equalization.

I. INTRODUCTION

This paper gives some quantitative estimates of the delay distortion due to mode conversion to be expected in multimode transmission lines such as the millimeter-wave circular electric waveguides. In principle, the results apply to any multimode system, including optical guided-wave systems; in the latter case, however, it is likely that the magnitude of delay distortion will be too small to be a limitation in practice. In millimeter waveguide systems, delay distortion effects can be important.

A very simple analytical approach is used, based on modes coupled through power-transfer coefficients. This permits a direct translation of the spatial distribution of mode conversion into the effect on the output waveform without recourse to the frequency domain as an intermediate step. The case treated herein is mode conversion that is an independent random function of distance, but the physical reasoning employed can be used to indicate the changes which alterations in the conversion distribution would cause. A perturbation method is employed, and limits are found for the distance for which this perturbation calculation is valid.

In a companion paper Dale T. Young[1] has shown how the power-transfer coefficients of the coupled-mode equations used herein are related to the more familiar coefficients between the amplitudes of coupled modes; this makes more quantitative the relation between the present work and the work of H. E. Rowe and W. D. Warters.[2] In another companion paper, L. H. Enloe[3] presents a precise technique for

analyzing the delay distortion effects due to mode conversion, starting with the work of Rowe and Warters[2] and D. T. Young;[4] additionally, Enloe has extended the work of Bennett, Curtis, and Rice[5] to facilitate the calculation of interchannel interference in FM systems in the presence of the exponential echo which is characteristic of mode conversion and reconversion effects.

Two different systems situations will be included in the numerical evaluations: (*i*) analog systems such as FM, wherein the important transmission line length is large, perhaps 4,000 miles, and (*ii*) regenerative systems, PCM, wherein the pertinent line length is the distance between regenerators, perhaps 15–50 miles.

Previous consideration has been given to FM on waveguide by Kazuhiro Miyauchi[6] and R. Hamer.[7] The present work seeks to add to our understanding by using impulse excitation and a simple physical model to clarify the nature of the distortion produced by mode conversion-reconversion effects and to make evident the controlling parameters. By using experimental results in combination with analysis, it turns out to be possible to define in a broad way the system capabilities of existing waveguides without specifying many of the exact waveguide tolerances. Also presented is a first treatment of the delay distortion effects in smooth-walled waveguides with partially absorbing mode filters inserted periodically.

II. PRELIMINARY FORMULATION OF THE PROBLEM

To evaluate delay distortion we assume the transmission of an impulse in the desired signal mode. Power will be converted to an undesired mode and reconverted back to the signal mode after some relative delay or advancement. Use will be made of the three-mode system previously employed[8] and sketched in Fig. 1. We designate the signal power as

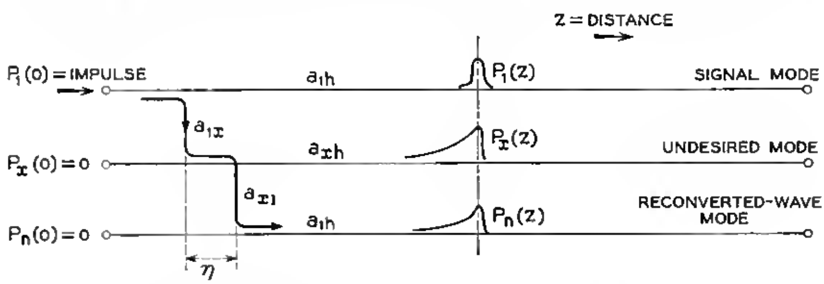

Fig. 1 — Diagram representing mode conversion and reconversion in multimode transmission systems.

$P_1(z)$, which decays exponentially as $e^{-(a_{1h}+a_{1x})z}$ due to heat loss a_{1h} and conversion loss a_{1x} coefficients. The undesired mode power $P_x(z)$ has a heat loss coefficient a_{xh} and a conversion loss coefficient a_{x1}. The third mode $P_n(z)$ contains the reconverted power, and has a heat loss coefficient a_{1h} and conversion loss coefficient a_{1x}. In Ref. 1 free conversion and reconversion is permitted between mode x and mode n, while conversion out of P_1 is permitted but no reconversion back from mode x to mode 1 takes place. Thus the signal mode total power is $(P_1 + P_n)$, but it is convenient to separate P_1 and P_n to identify the power which has undergone conversion at least once (P_n).

In the present pulse analysis we will employ only one mode conversion-reconversion sequence and identify the limits of applicability of this approximation.

We use power flow directly, as in the appendix of Ref. 8, and assume continuous random coupling between the modes. From this we get the expected value for the reconverted power under the conditions assumed; this is a useful first step, although more detailed knowledge would be desirable.

The key to this analysis is the fact that the converted pulse in the x mode suffers simultaneous delay and attenuation relative to the signal mode. No matter where in the line a component is converted and reconverted, a relative delay τ is always associated with a relative attenuation $e^{-(a_{xh}-a_{1h})\eta}$, where η is the distance the power travels in the x mode. Also,

$$\tau = \frac{\eta}{v_x} - \frac{\eta}{v_1} \tag{1}$$

where v_x and v_1 are the group velocities in mode x and mode 1 respectively.

The magnitude of the reconverted power density P_nD relative to the signal pulse P_1 is (for $z >$ a minimum distance to be specified)

$$P_nD(\eta) = K \exp\left[-(a_{xh} - a_{1h})\eta\right] \tag{2}$$

where K is to be determined. By using (1) we can replace η in (2) by τ, giving the reconverted power density versus τ (Fig. 2). We see immediately that the echo power due to conversion and reconversion varies as $e^{-\tau/\tau_0}$, where τ_0 depends only on the group velocities and the heat loss coefficients. Equation (2) can be integrated over all values of η, the distance travelled in the x mode, and equated to the average reconverted power P_n to evaluate K.

$$\int_0^\infty P_nD(\eta)\, d\eta = \text{average power in mode } n. \tag{3}$$

The next section establishes the right-hand side of (3).

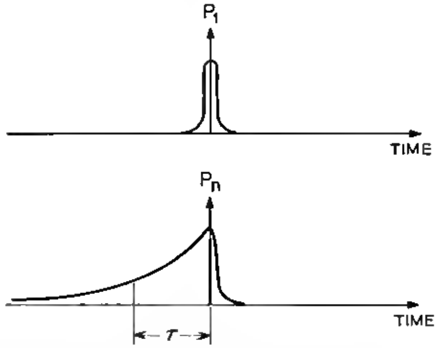

FIG. 2 — Signal impulse (P_1) and reconversion echo wave (P_n) versus time after transmission through a multimode line with mode conversion.

III. DETERMINATION OF THE AVERAGE RECONVERTED POWER

We use here the results of an earlier analysis.[8] We assume continuous excitation of the signal mode at the sending end, and write the following equations for power flow between modes

$$\frac{dP_1}{dz} = -(a_{1h} + a_{1x})P_1$$

$$\frac{dP_x}{dz} = -(a_{xh} + a_{x1})P_x + a_{1x}P_1 + a_{1x}P_n$$

$$\frac{dP_n}{dz} = -(a_{1h} + a_{1x})P_n + a_{x1}P_x .$$

In Ref. 8 these equations are solved for $P_1(z = 0) = 1$ and $P_x(z = 0) = P_n(z = 0) = 0$; dividing that work's equation (21) by equation (19) we obtain

$$\frac{P_n}{P_1} = -1 + \frac{1}{\surd}\left\{\frac{1}{2}[-(\alpha - \beta) + \surd] \exp [\tfrac{1}{2}[-(\beta - \alpha) + \surd]z] - \frac{1}{2}[-(\alpha - \beta) - \surd] \exp [\tfrac{1}{2}[-(\beta - \alpha) - \surd]z]\right\} \tag{4}$$

where

$$\alpha = a_{1h} + a_{1x}$$

$$\beta = a_{xh} + a_{x1}$$

$$\surd = \sqrt{(\alpha - \beta)^2 + 4a_{1x}a_{x1}} .$$

It can be verified that P_n/P_1 grows without limit for $z \to \infty$ because the exponent $[-(\beta - \alpha) + \sqrt{\ }]$ is positive. This is an expected result due to the fact that conversion and reconversion take place between mode x and mode n, whereas power converts one way from mode 1 to mode x.

We now seek to limit z in such a way that (4) is small compared to unity. We assume

$$a_{1x} = a_{x1} \tag{5}$$

$$a_{1x}^2 \ll (a_{xh} - a_{1h})^2. \tag{6}$$

Then (4) reduces to

$$\begin{aligned} \frac{P_n}{P_1} = -1 + \exp\left[a_{1x}^2 z/(a_{xh} - a_{1h})\right] + \frac{a_{1x}^2}{(a_{xh} - a_{1h})^2} \cdot \\ \exp\left[-(a_{xh} - a_{1h})z\right] - \frac{a_{1x}^2}{(a_{xh} - a_{1h})^2} \exp\left[a_{1x}^2 z/(a_{xh} - a_{1h})\right] \end{aligned} \tag{7}$$

Now, by requiring

$$(a_{xh} - a_{1h})z \gg 1 \tag{8}$$

we drop the third term of (7), and by letting

$$\frac{a_{1x}^2 z}{(a_{xh} - a_{1h})} \ll 1 \tag{9}$$

we can expand the second term of (7) (noting the fourth term is negligible compared to the second), yielding

$$\frac{P_n}{P_1} = \frac{a_{1x}^2 z}{(a_{xh} - a_{1h})}\,. \tag{10}$$

Conditions (6) and (8) are not restrictive on the maximum length z of the system and are very typical. Condition (9) is restrictive on z, and by requiring it to be true, we discover P_n/P_1 has the same form. Hence, so long as P_n/P_1 calculated from (10) is small compared to unity, it is a valid calculation. Numerical values will be inserted at a later point.

IV. JUSTIFICATION OF FIRST-ORDER PERTURBATION SOLUTION

We now consider the validity of using only the first conversion and reconversion term. Using equations (19), (20) and (21) of the appendix to Ref. 8, it can be shown that when restriction (8) of this paper is met

it is also true that

$$\frac{P_1 + P_n}{P_x} = \frac{(a_{xh} - a_{1h})}{a_{1x}} \tag{11}$$

which is independent of z. We will see that this is typically a number of the order of 100 to 1000 or more. As stated above, when (9) is also true, $P_n \ll P_1$, and hence

$$\frac{P_1}{P_x} = \frac{a_{xh} - a_{1h}}{a_{1x}}. \tag{12}$$

This is believed to be important. P_1 decays simply as $\exp[-(a_{1h} + a_{1x})z]$ independently of all reconversion effects; P_1 therefore serves as a convenient reference, and the other powers P_x and P_n are normalized with respect to it. For any z [subject to (8) and (9)] P_1/P_x is a constant. With increasing z, power is being converted continuously from P_1 to P_x and from P_n to P_x, but P_1/P_x is constant despite the fact that P_n/P_1 grows through orders of magnitude. It must follow that negligible power in mode x comes from mode n in the range for which (9) is valid. Since negligible power in mode x was ever in mode n, the second- and higher-order processes of conversion and reconversion can be ignored.

V. RECONVERSION POWER DENSITY VERSUS RELATIVE DELAY

Equation (3) can now be written [using (10)]

$$\int_0^\infty K \exp[-(a_{xh} - a_{1h})\eta]\, d\eta = \frac{a_{1x}^2 z}{(a_{xh} - a_{1h})} P_1. \tag{13}$$

Performing the integration to evaluate K and substituting back into (2) gives

$$P_n D(\eta) = P_1 a_{1x}^2 z \exp[-(a_{xh} - a_{1h})\eta] \tag{14}$$

or, using (1) to eliminate η in (13)

$$P_n D(\tau) = \frac{P_1 a_{1x}^2 z}{\left(\frac{1}{v_x} - \frac{1}{v_1}\right)} \exp\left[-(a_{xh} - a_{1h})\tau \Big/ \left(\frac{1}{v_x} - \frac{1}{v_1}\right)\right]. \tag{15}$$

In either case we have assured that

$$\int_0^\infty P_n D(\tau)\, d\tau = \int_0^\infty P_n D(\eta) d\eta = \frac{P_1 a_{1x}^2 z}{(a_{xh} - a_{1h})}. \tag{16}$$

It is convenient to define a time constant τ_0 at which the reconversion

power density is down to $1/e$ times the $\tau = 0$ value

$$\tau_0 = \frac{\left(\frac{1}{v_x} - \frac{1}{v_i}\right)}{(a_{xh} - a_{1h})} \approx \frac{1}{2c} \frac{(\nu_x^2 - \nu_1^2)}{(a_{xh} - a_{1h})} \tag{17}$$

where

c = velocity of light in the dielectric in the waveguide
ν_y = $\lambda k_y / \pi d$
d = waveguide diameter
k_y = Bessel root appropriate to the cutoff of mode y, and
λ = intrinsic wavelength characteristic of the dielectric in the waveguide.

The part of (17) involving ν's instead of group velocities v assumes the waveguides are far enough from cutoff that

$$\sqrt{1 - \nu^2} \approx 1 - \frac{1}{2}\nu^2.$$

In some system applications it may be acceptable to ignore reconversion tails closer to the signal pulse than some time interval τ_r. It is of interest then to sum the reconverted powers for τ from τ_r to ∞, which is the shaded area in Fig. 3. From (15)

$$P_{nr} = \int_{\tau_r}^{\infty} P_n D(\tau)\, d\tau = \frac{P_1 a_{1x}^2 z}{(a_{xh} - a_{1h})} \exp\left[-\tau_r/\tau_0\right]. \tag{18}$$

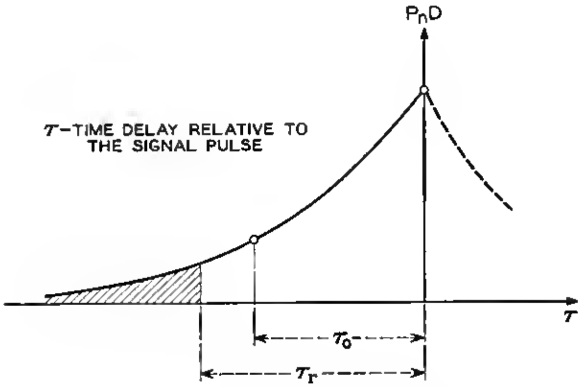

Fig. 3 — Density of reconverted wave power (P_nD) versus time when there is coupling between modes with distributed differential loss.

VI. EFFECT OF MODE FILTERS ON DELAY DISTORTION

The preceding paragraphs cover the case of distributed differential loss. When the undesired mode loss is lumped, as is the case when mode filters are used, (1) still holds, but we need a new expression to replace (2) to represent the density of reconverted power as a function of the distance η that the power traveled in the undesired mode. The appropriate relation is

$$\begin{aligned} P_n D(\eta) &= A\left[1 - \frac{\eta}{s} + \frac{C_1\eta}{s}\right] \text{ for } 0 < \eta < s \\ &= A\left[C_1\left(2 - \frac{\eta}{s}\right) + C_1^2\left(\frac{\eta}{s} - 1\right)\right] s < \eta < 2s \qquad (19) \\ &= A\left[C_1^2\left(3 - \frac{\eta}{s}\right) + C_1^3\left(\frac{\eta}{s} - 2\right)\right] 2s < \eta < 3s \end{aligned}$$

in which s is the mode filter spacing, C_1 is the power transmission coefficient for the mode filters, and the number of mode filter sections n must be large compared to $(p + 1)$, where $ps < \eta < (p + 1)s$. The proportionality constant A is analogous to K of (2), and is to be evaluated by equating the integral of $P_n D(\eta)$ over all η to the average reconverted power P_n. To do so we evaluate P_n (assuming $a_{1x} = a_{x1}$)

$$\begin{aligned} P_n = \int_0^{ns} a_{1x} P_x(y)\, dy = n \int_0^s a_{1x} P_x(y)\, dy + (n - 1) \\ \int_s^{2s} a_{1x} C_1 P_x(s)\, dy + (n - 2) \int_{2s}^{3s} a_{1x} C_1^2 P_x(s)\, dy + \cdots \end{aligned} \qquad (20)$$

in which y is the distance in the direction of propagation, analogous to z of Fig. 1. We use a loose coupling approximation and assume random coupling effects as above. We can assume the loss in the driven mode (no. 1) is small in the length s between mode filters, so that we can write $P_x(y)$ in the interval $ps < y < (p + 1)s$, p any integer,

$$P_x(y) = P_1(ps) \int_0^{(y-ps)} a_{1x}\, dy + C_1 P_x(ps). \qquad (21)$$

We evaluate (20) using the approximation that there is little difference in attenuation (other than the mode filter loss) between modes 1 and x in the length η which is ultimately important

$$P_n = P_1 \frac{n a_{1x}^2 s^2}{2} \left[1 + \frac{2(n - 1)C_1}{n} + \frac{2(n - 2)C_1^2}{n} + \cdots\right]. \qquad (22)$$

For $n \gg k$, where C_1^k is the highest-order term of importance and $z = ns =$ total length of line

$$P_n \approx \frac{a_{1x}^2 sz}{2} \frac{(1 + C_1)}{(1 - C_1)} P_1 . \tag{23}$$

This expression is analogous to (10) and P_1 is again the power at z in the driven mode. We now evaluate A of (19) by equating the two expressions for P_n

$$P_n = P_1 \frac{a_{1x}^2\, sz}{2} \frac{(1 + C_1)}{(1 - C_1)} = \int_0^\infty P_n D(\eta)\, d\eta = \frac{s}{\tau_s} \int_0^\infty P_n D(\tau)\, d\tau \tag{24}$$

where

$$\frac{\eta}{s} = \frac{\tau}{\tau_s} \tag{25}$$

$$\tau_s = s \left(\frac{1}{v_x} - \frac{1}{v_1} \right) \tag{26}$$

$$\approx \frac{s}{c} \left(\frac{1}{2} \nu_x^2 - \frac{1}{2} \nu_1^2 \right).$$

Using (19) for $P_n D(\eta)$ we perform the integration of (24) and find, approximately

$$A = a_{1x}^2 z P_1 . \tag{27}$$

We can now put (19) in final form

$$\begin{aligned} P_n D(\tau) &= P_1 a_{1x}^2 z \left[1 - \frac{\tau}{\tau_s} + \frac{C_1 \tau}{\tau_s} \right] 0 \leqq \tau \leqq \tau_s \\ &= P_1 a_{1x}^2 z \left[C_1 \left(2 - \frac{\tau}{\tau_s} \right) + C_1^2 \left(\frac{\tau}{\tau_s} - 1 \right) \right] \\ &\quad \tau_s \leqq \tau \leqq 2\tau_s \\ &= P_1 a_{1x}^2 z \left[C_1^2 \left(3 - \frac{\tau}{\tau_s} \right) + C_1^3 \left(\frac{\tau}{\tau_s} - 2 \right) \right] \\ &\quad 2\tau_s \leqq \tau \leqq 3\tau_s \end{aligned} \tag{28}$$

or more generally

$$P_n D(\tau) = P_1 a_{1x}^2 z \left[C_1^k \left(k + 1 - \frac{\tau}{\tau_s} \right) + C_1^{k+1} \left(\frac{\tau}{\tau_s} - k \right) \right] \qquad k\tau_s \leqq \tau \leqq (k+1)\,\tau_s \tag{28a}$$

and $k = 0, 1, 2, 3, 4$, etc. This function is plotted in Fig. 4. The reconversion power density declines linearly from its $\tau = 0$ value to C_1 times its $\tau = 0$ value at $\tau = \tau_s$. For $\tau_s < \tau < 2\tau_s$ the reconversion power density again declines linearly from $C_1 a_{1x}^2 z P_1$ to $C_1^2 a_{1x}^2 z P_1$, and similarly for larger τ.

It is again of interest to evaluate the total reconversion power from $\tau = \tau_r$ to $\tau = \infty$ because echoes at very short time delays are not damaging. The ratio of reconversion power for $\tau > \tau_s$ to the total reconversion power may be shown to be

$$\int_s^\infty P_n D(\eta)\, d\eta = P_n - \int_0^s P_n D(\eta)\, d\eta = C_1 P_n \tag{29}$$

and similarly for reconverted power at $\tau > 2\tau_s$

$$\int_{2s}^\infty P_n D(\eta)\, d\eta = \int_s^\infty P_n D(\eta)\, d\eta - \int_s^{2s} P_n D(\eta)\, d\eta = C_1^2 P_n . \tag{30}$$

Thus for $\tau_r = m\tau_s$ the total reconverted power from $\tau = \tau_r$ to $\tau = \infty$ declines as C_1^m.

VII. DISTANCES FOR WHICH THE APPROXIMATIONS HOLD

Two approximations concerning z have been made, relations (8) and (9). The minimum z comes from (8). We calculate for three waveguide cases of interest

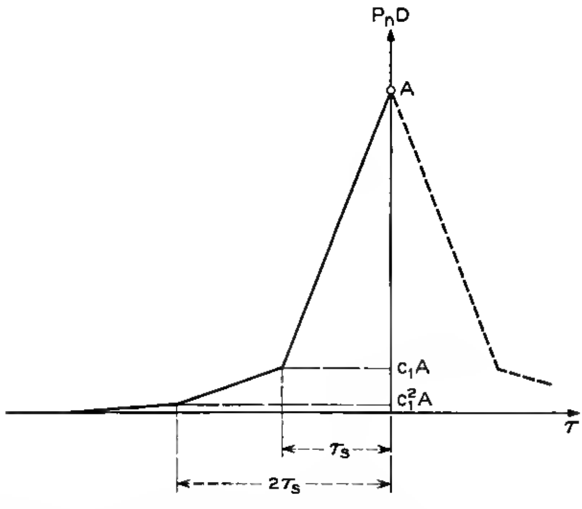

Fig. 4 — Density of reconverted wave power ($P_n D$) versus time when there is coupling between modes with lumped differential loss.

(*i*) helix damped modes in a 2-inch I.D. all-helix waveguide,

(*ii*) modes (such as TE_{02}) which are attenuated only by the normal copper losses in the walls; this situation is the same in helix waveguide or in solid-walled copper waveguide, where again we use the 2-inch I.D. case, and

(*iii*) 2-inch I.D. solid-walled copper guide with a 15-foot helix mode filter every 300 feet.

Condition (8) appears because we want to make $\exp[-(a_{xh} - a_{1h})z]$ negligible compared to unity; we will use as a criterion for minimum z ($z_{\min}$) that $(a_{xh} - a_{1h})\, z_{\min} = 3$. The table below gives the typical values for $(a_{xh} - a_{1h})$ and the resulting $z_{\min}$. For case (*iii*), $z_{\min} \gg ms$, where m is chosen so that $C_1^m \ll 1$; this assures that the line-segment exponential reconversion trail (Fig. 4) is fully developed. Except possibly for case (*ii*) these restrictions are not very limiting.

The maximum $z(z_{\max})$ for which these estimates are valid comes from (9) and (10) or (23); we take $P_n/P_1 = 0.1$ as the definition of $z_{\max}$, although somewhat larger z's might give meaningful results. The values of $(a_{xh} - a_{1h})$ from Table I (known from measurements on physical waveguides) are again used. The values of conversion coefficient a_{1x} are known from measurements on experimental waveguides at the Holmdel, N. J., Bell Laboratories for cases (*i*) and (*iii*) but are not known from measurement for case (*ii*). For the latter the theory of H. E. Rowe and W. D. Warters[2] was used, in combination with known data on the diameter variations of our good waveguides; a 1-mil rms diameter variation was taken, and the Warters-Rowe calculation for the average loss for random discrete imperfections on 10-foot joint spacing used to get the case (*ii*) value of a_{1x} in Table II. These are conservative values when applied to the 55-kmc region; Holmdel waveguides have shown comparable conversion coefficients at higher frequencies.

For all-helix waveguide, it appears that these estimates should be good for a cross-country analog system; for copper tubing with mode filters the estimates are good for 100 miles or thereabouts.

TABLE I

Waveguide Case	Amplitude Decay Coefficient $(a_{xh} - a_{1h})/2$*	$(a_{xh} - a_{1h})$	$z_{\min}$
(*i*)	1 db/foot	0.23/ft.	13 feet
(*ii*)	4.3 db/mile	$\approx$1/mile	3 miles
(*iii*)			3000 feet

* With the necessary conversion factors to give the units shown.

Table II

Waveguide Case	Average Amplitude Conversion Loss $a_{1x}/2$*	$a_{1x}/(a_{xh} - a_{1h})$	a_{1x}	z_{max} miles
(i)	1 db/mile	1/5280	0.23/mile	2290
(ii)	0.005 db/mile	1/860	0.00115 per mile	74,600
(iii)	1 db/mile		0.23/mile	62

* With the necessary conversion factors to give the units shown.

VIII. NUMERICAL VALUES FOR RECONVERSION TIME CONSTANTS

In this section we will evaluate the relations previously derived for the time constants of the reconverted powers, using parameters typical of millimeter-wave circular electric waveguides.

It should be noted that the reconverted power will precede as well as lag the signal impulse. Significant mode conversion takes place between TE_{01} and TE_{11} due to straightness variations, and the group velocity for TE_{11} is greater than that for TE_{01}. This will not be labored at greater length here; a *typical* group velocity difference will be calculated, and part of the reconverted power for straightness variations will precede the signal and part will lag the signal.

It should also be noted that the time variation of the reconversion power as given by (15) and (28) is independent of distance z within the limits $z \leq z_{max}$ calculated above. The magnitude of reconversion is of course dependent on z.

Equations (17) and (26) show that less delay occurs when the velocity difference is smaller, and this is in turn obtained when ν is smaller, i.e., the guide is farther from cutoff. From (17) or (26) the reconversion duration varies as $1/f^2d^2$ where d = waveguide diameter and f = frequency. The estimates given here are for 2-inch I.D. guide at 55 kmc.

Consider the delay distribution of mode conversion due to random unintentional straightness variations, which causes coupling to the TM_{11}, TE_{11}, and TE_{12} modes. Because TE_{12} gives the longest echo trail, we use it as the mode for the computation of $(\nu_x^2 - \nu_1^2)/2$ and evaluate (17) and (26) as shown in Table III.

Table III

Waveguide Case	$(\nu_x^2 - \nu_1^2/2$	Delay Evaluation
(i) (all-helix)	0.0079	$\tau_0 = 0.0348 \times 10^{-9}$ sec
(iii) (solid copper with mode filters)	0.0079	$\tau_s = 2.41 \times 10^{-9}$ sec

The other waveguide condition of interest, case (*ii*), either helix or solid-walled copper with circular electric mode conversion, gives an exponential reconversion shape; $(\nu_x^2 - \nu_1^2)/2 = 0.0197$ (for $TE_{01} - TE_{02}$), $(\alpha_{xh} - \alpha_{1h})$ corresponds to an attenuation difference of 4.3 db/mile, and $\tau_0 = 105.8 \times 10^{-9}$ second from (17).

IX. SYSTEM EVALUATION OF MODE CONVERSION DELAY DISTORTION

Some comments can be made on the relation between expected system requirements and reconversion power magnitude and delay distribution, even though further analysis in the case of analog systems will be necessary to get precise numbers.

9.1 *PCM on an All-Helix Waveguide Line*

Consider first an all-helix waveguide line. Typical values of conversion coefficient a_{1x} and heat-loss difference $(\alpha_{xh} - \alpha_{1h})$ have already been given in connection with Tables I and II. The first row of Table I applies for random straightness deviations, and the second row applies for diameter variations. If we require a total P_n/P_1 ratio (summed over all delay times) of -30 db (as might be desirable in a PCM system of infinite pulse rate) we find from (18) with $\tau_r = 0$ that a distance z of 22.9 miles is permissible as far as straightness variations are concerned, and $z = 746$ miles is permissible from the point of view of diameter variations. The shorter distance controls, and we can conclude for helix waveguide there is no limit on the pulse width permitted by reconversion delay distortion in a PCM system with regenerators every 20 miles. We can go further, however. For reconversion delays very short compared to the pulse rate there will be no interference between time slots; the reconverted power shows as a variation in apparent loss in the line and provided that this loss shift is stable, the reconverted power can be relatively large. Suppose we let $z = 4000$ miles; then the total $P_n/P_1 = -7.6$ db with respect to straightness variations, and the total $P_n/P_1 = -22.6$ db with respect to diameter variations. Using (18), we can compute the τ_r for the total P_{nr}/P_1 in the range $\tau_r < \tau < \infty$ to be -30 db; we find $\tau_r/\tau_0 = 5.16$ or $\tau_r = 0.18 \times 10^{-9}$ second with respect to straightness variations, and $\tau_r/\tau_0 = 1.7$ or $\tau_r = 180 \times 10^{-9}$ second with respect to diameter variations. This implies that straightness variations (or other effect causing conversion to modes damped by the helix) do not limit and diameter variations do limit the length between regenerators in PCM systems with all-helix waveguide. By backing off from $z = 4000$ to $z = 746$ miles, the desired total $P_n/P_1 = -30$ db

including reconverted power at all time delays; thus satisfactory PCM operation with regeneration every 746 miles is permitted by reconversion delay effects in an all-helix waveguide line for pulse rates up to about $10^9/0.18$ or 5000 megabits/second.

9.2 *FDM-FM on an All-Helix Waveguide*

The work of Bennett, Curtis, and Rice[5] allows us to estimate the limits of transmitting a single-sideband group of telephone channels by FM (FDM-FM) as is done on the TD-2 and TH radio relay systems. Their work shows that the RF signal-to-echo ratio need not be as large as the interchannel interference requirement, and since the length of the permitted system is inversely proportional to the required RF signal-to-echo power ratio (Ref. 5, equation 18) this can greatly change the permissible system length. The work of Bennett et al.[5] (their Fig. 5.7) shows that when the ratio σ/f_b, (rms frequency deviation)/(highest baseband frequency), is unity or more the requirements on reconverted power are essentially independent of delay for $\tau > 0.2/f_b$ (f_b = highest baseband frequency) and becomes very rapidly more tolerant of reconverted power for smaller delays. At $\sigma/f_b = 1$ an FM advantage of about 7 db for large time delays is given by Bennett et al., and for larger σ/f_b the FM advantage increases at the rate of 8.4 db/octave.* It seems reasonable to assume an FM advantage of at least 10 db, and more probably could be realized. This means that for interchannel interference of -40 db, $P_{nr}/P_1 = -30$ db can be used, and since the requirement falls off rapidly below $\tau f_b = 0.2$, an assumed 10-mc SSB baseband signal (2000 channels) allows us to use $\tau_r = 20 \times 10^{-9}$ seconds in computing the permissible system length from (18). The resulting estimate is 900 miles, maximum system length for a 10-mc group of SSB-AM channels to have -40 db interchannel interference ratio when sent with an FM index σ/f_b of about $\sqrt{2}$ on an all-helix waveguide described by cases (*i*) and (*ii*) of Tables I and II. (The limitation is TE_{02}.) Mr. L. H. Enloe has made a much more rigorous analysis and found a similar result.[3]

Since the FDM-FM application is limited only by conversion to higher circular electric modes, there would be considerable advantage in adding mode filters for such modes.[6] This is certainly possible in principle but has not been reduced to practice. Estimates of requirements for circular electric mode filters can be deduced from (23), (26) and (28).

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* This and the succeeding discussion mean that the FM advantage and the allowed system length vary as the 2.8 power of the frequency deviation when long-delay echoes are controlling.

9.3 *PCM on Solid-Walled Copper Waveguide*

Turning now to PCM transmission in solid-walled copper waveguide with (noncircular electric) mode filters, the circular electric mode conversion limitations are the same as in an all-helix waveguide line. Typical conversion loss and heat loss numbers are given in case (*iii*) of Tables I and II. For straightness variations (modes damped by the mode filters) and for $P_n/P_1 = -30$ db including all time delays, the permissible distance between regenerators is 0.62 mile for 300-foot mode filter spacing; P_1/P_n, and hence permissible system length, varies inversely as the mode filter spacings. This form of line will *not* handle indefinitely large PCM pulse rates for 15–20-mile repeater spacings. For 300-foot mode filter spacing the length of reconversion pulse (τ_s in Fig. 4) is $\tau_s = 2.5 \times 10^{-9}$ second (per Table III); a PCM system using a pulse rate of 200 megabits would probably be acceptable up to distances at which the "second" ($\tau_s < \tau < 2\tau_s$) reconversion pulse becomes limiting. For 15-db mode filters,* this would be approximately a distance z such that

$$10 \log_{10} (z/0.62) = 15 \text{ db}$$

or $z \approx 20$ miles between regenerators. For operation where the third ($2\tau_s < \tau < 3\tau_s$) reconversion interval is controlling — i.e., at PCM rates of about 100 megabits — the length z can be

$$10 \log (z/0.62) = 30 \text{ db}$$

or $z = 620$ miles† between regenerators. Alternatively, by using mode filters spaced 150 feet instead of 300 feet, the permissible pulse rate could be 400 megabits at 40 miles. We found above that diameter variations permit a length between regenerators ($P_1/P_n = -30$ db) of 746 miles, and hence this imperfection is not limiting for high-speed PCM in solid-walled guide with mode filters.

9.4 *FDM-FM on Smooth-Walled Copper Waveguide*

For the same guide with FDM-FM transmission where we may require [in (23)] $P_n/P_1 = -30$ db as above, the length z permitted by straightness variations is 0.62 mile (for all delay times considered interfering). If we ask what τ_r would make the permitted system length controlled by straightness variations equal to 900 miles, the figure found

* A 15-foot section of 2-inch I.D. Holmdel helix would have 15 db loss to TE_{12} and TE_{11} at 55 kmc and even more loss for TM_{11}, the other significant mode.

† Here we have run beyond the 100-mile limit of applicability of the perturbation theory and the result needs further justification.

permissible due to diameter variations, we find [as indicated in the discussion associated with (29) and (30)]

$$C_1^{\tau_r/\tau_s} = \frac{0.62}{900} .$$

Letting $C_1 = 0.0316$ (corresponding to a 15-db mode filter), we find $\tau_r/\tau_s \approx 2$ and $\tau_r \approx 5 \times 10^{-9}$ sec. Neglecting echoes with time delays less than 5×10^{-9} seconds is compatible with tbe assumptions made above for the FDM-FM transmission and appears to be a real possibility. However, to go to 900 miles system length is in violation of the $z_{\max} =$ 62 miles computed earlier, and indeed it is clear that the perturbation calculation is no longer valid. We might inquire wbat would be tbe practical effect of longer systems than that for which the perturbation calculation holds.* When all of the power in the driven mode has been reconverted once, the driving signal would no longer be an impulse but might roughly be a waveform of the character sketched in Fig. 4. Then further conversion and reconversion would broaden this waveform again. As a guess we might expect one perturbation-theory time-constant of delay distortion for every distance such that $P_n/P_1 \approx 1$. For the case being considered, $P_n/P_1 \approx 1$ at $z = 620$ miles, and in this vicinity the above calculation showed a τ_r of 5×10^{-9} seconds would be adequate to meet requirements; if we now allow $\tau_r = 20 \times 10^{-9}$ seconds as done above ($\tau f_b \approx 0.2$) we might expect to tolerate $(20/5) \approx 4$ times 600 miles system length. Thus, even in solid-walled guide with mode filters the reconversion power may be concentrated at such small delays as to permit FDM-FM for 900 miles and possible greater distances. This must be regarded as a tentative estimate, a major uncertainty being the nature of the delay distortion in a multimode system in the region where several successive conversions and reconversions must be accounted for (i.e., perturbation theory not valid).

* Another interesting question concerns the way echo power will be related to a possible "breaking" phenomenon at the FM demodulator. This is quite different from the breaking which occurs due to thermal noise. For echo delays which are short compared to the reciprocal of the modulating baseband frequency, the writer has concluded that no appreciable fluctuation in the envelope results, and the echo power has the principal effect of causing a static shift in carrier magnitude which would differ among members of an ensemble of waveguide lines but which would be approximately fixed (varying perhaps slowly due to temperature effects, drifts in carrier frequency, etc.) in any one line. For echo delays which are comparable to the reciprocal of the modulating baseband frequency, envelope fluctuations do result and limiters can be used to advantage. Since some waveguide modes produce reconversion echoes which occur predominantly at delays short compared to the modulating periods, the total tolerable reconversion power may be significantly greater than the tolerable thermal noise at the FM demodulator. Further study of specific situations is needed to answer the question.

In practice there is another factor which will make solid-walled waveguide less speculative for FDM-FM. As L. C. Tillotson has pointed out, one can demodulate the signal often enough so that the interference calculated from perturbation theory is valid. This is more than a ruse. By demodulating and filtering, the interchannel interference terms resulting from very short delay echoes are discarded. After remodulating, the short delay echoes are missing and hence they cannot go through successive steps of delay distortion to become longer delay echoes in the manner discussed above. Thus by demodulating at approximately the intervals at which the perturbation theory breaks down (100 miles approximately) one would filter out the short delay echoes and prevent their ever causing crosstalk. Since one usually drops and adds channels at intervals of 100–200 miles, it is possible that the demodulation procedure would not be a serious limitation in many cases.

X. CONCLUSIONS AND DISCUSSION

Using a simple power-flow analysis, expressions have been found for the magnitude and delay distribution for the expected value of the reconverted power in a multimode transmission system with mode conversion. Three cases are examined in detail:

(*i*) the helix-damped modes in an all-helix line,

(*ii*) circular electric mode conversions in either helix or smooth-walled guide, and

(*iii*) the mode filter damped modes in a line made up of smooth-walled copper plus mode filters.

For numerical examples, 2-inch I.D. guides at 55 kmc are assumed. The minimum and maximum lengths of line (between terminals in an analog system or between regenerators in a PCM system) for which the analysis is valid are given in Tables I and II respectively.

For impulse excitation, the reconversion echo as a function of delay τ relative to the signal pulse varies as $e^{-\tau/\tau_0}$ as given in Fig. 3 for cases (*i*) and (*ii*), and varies as a line segment approximation to an exponential illustrated in Fig. 4 for case (*iii*). For mode conversion due to straightness variations (TE_{12}), τ_0 in helix waveguide is about 0.035 nanosecond and τ_s (Fig. 4) in copper guide with mode filters spaced 300 ft. is about 2.4 nanoseconds. For diameter variations (TE_{02}) in either type of guide τ_0 is about 106 nanoseconds; this could be radically reduced by the addition of circular electric mode filters, but system estimates herein do not count on that, since such mode filters have not yet been developed.

Only the echo power at delays greater than some relative delay τ_r is subject to severe requirements in typical system layouts, and $\tau_r \gg \tau_0$ for certain imperfections in either helix waveguide or smooth-walled waveguide. The reconverted power P_n, relative to signal power P_1 at the same distance z, for $\tau_r < \tau < \infty$ is given in (18) for cases (*i*) and (*ii*)

$$P_{nr} = \int_{\tau_r}^{\infty} P_n D(\tau)\, d\tau = \frac{P_1 {a_{1x}}^2 z}{(a_{xh} - a_{1h})} \exp\left(-\tau_r/\tau_0\right) \qquad (18)$$

where a_{1x} is the average mode conversion coefficient and a_{xh} and a_{1h} are the heat loss coefficients in the undesired mode and signal modes respectively. Since reconverted power at delays less than some value τ_r will usually have negligible system effect, (18) evaluates the important reconversion power. For case (*iii*) similar results are given by (28), (29) and (30).

For the transmission of PCM on an all-helix line, the discussion of Section 9.1 indicates that diameter variations limit the regenerator spacing to about 746 miles and random straightness variations limit the pulse rate to 5000×10^6 per second. For analog transmission an FM advantage on the echo interference as computed for a single echo by Bennett, Curtis and Rice[5] is essential to get reasonable requirements on the RF signal-to-echo ratio. (See Section 9.2.) Using the theory of Bennett et al., a rough estimate indicates that 2000 channels could be sent a distance of 900 miles using an rms frequency deviation of $\sqrt{2}$ times the top baseband frequency, 10 mc; larger deviations would yield longer systems at a rate 6.9 times the system length for 2:1 increase in deviation. The length limit is again set by diameter variations. A more precise technique for computing the interchannel interference due to the exponential echo trails which are characteristic of multimode systems, Figs. 2 and 3, is given by L. H. Enloe in a companion paper.[3]

For the transmission of PCM on smooth-walled guide plus mode filters, the discussion of Section 9.3 indicates that pulse rates of 200 or 400 megabits with mode filters spaced 300 feet or 150 feet might be used with distances up to 20 or 40 miles respectively between regenerators. Longer distances between regenerators are possible if the bit rate or the mode filter spacing is reduced. In Section 9.4, analog transmission of the same FDM-FM signal noted above (2000 channels) may be possible in smooth-walled guide with mode filters for the same system lengths and deviation ratio noted above.

All of these estimates are very sensitive to the magnitude of the controlling imperfections in the medium; the permissible system length varies inversely as the fourth power of the geometric distortion magni-

tude. The echo duration estimates are also made for 2-inch guide at 55 kmc, and vary inversely as (frequency)^2 for fixed heat-loss coefficients. In practice, differential heat loss decreases with increasing frequency, tending to reduce the broad-range frequency variation in echo duration.

Separate consideration is being given to delay distortion due to waveguide cutoff dispersion, which will be appreciable in some system configurations described and will require equalization. Equalization of the delay distortion due to mode conversion is also being considered separately. It is interesting to note that each delay difference τ [of (28) or (15)] corresponds to a unique value of η, the distance the energy travelled in the undesired mode, and that the phase difference between all reconverted components and the unconverted (or "straight-through") signal component is also unique for a given τ. The random magnitudes (but not phase angles) of the conversion coefficients and the random location along the axis of propagation may make this surprising at first. There may be hope of equalizing out the expected value of the reconversion echo, but we must keep in mind that the variance of the echo magnitude will limit the degree to which this is possible. In this paper the system estimates are based on no equalization of the mode conversion effects.

A constructive discussion of FM with L. C. Tillotson, C. L. Ruthroff, and L. H. Enloe is gratefully acknowledged.

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